Understanding Least Squares Adjustments

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Richard Maher, PLS – KDM Meridian
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- Alternate Director and Tech Czar for OC-CLSA
- Current Chair of CSRC Executive Committee
- SoCal Engineering/Surveying Firm 10 years
  - Owned/Operated KDM Meridian 15 years
  - 10 Years of Least Squares Analysis and Adjustment experience with StarNet and 5 years prior experience with TGO/TBC
Outline: Least Squares Analysis and Adjustment (LSA)

- What LSA is
- Why we want to use LSA
- How we can use LSA
- Moving forward with LSA
What LSA is

- Not new…
  - 200 years / Carl Guass / Astronomy
  - Bell Curve “Guassian Distribution”
  - Surveying just one field applied to
- Accepts reality of random error, and mistakes
- Distributes random error properly (by weight)
- Tool for identifying mistakes (blunders)
- Planning tool for achieving statistical accuracies
What LSA is

- The principle of least squares states that: The sum of the squares of the residuals—multiplied by their appropriate weights—will be a minimum for that set of answers that has the greatest probability of being correct.

- Can be simple or complex: applies to computing a simple mean of two equally weighted measurements—or- mixing various methods and equipment in one survey
What LSA is

- Difficult to nearing impossibility without computers
- Modern computers bring it within capability of all
- Available in nearly all surveying software
- Ingrained in the DNA of several of our tools (GPS Receivers, Data Collectors, Digital Levels)

Our challenge is to set up the problem correctly and interpret the results, with iteration
What LSA is

- Best geometrical result, given valid input
- Solution is iterative, “converging” when the next successive adjustments are smaller than the previous
- Standard deviations allowing calculations of error propagation
- Rigorous and defensible, not a guess
Why we want to use LSA

- LSA will assist you in arriving at the best answer (probable truth & accuracy) in your survey.
- I believe so strongly that I apply it to every survey, every measurement, in my practice.
- Evidenced that many rely on the precision of their instruments and blend the data together through various means.
Why we want to use LSA

- GPS (static and RTK/N, Digital Levels, Digital Data Collection: Information (answers) Overload
- Simple Compass, Crandall, Level Adjustments?
- Overly determined surveys(networks) are what LSA is made for.
- Statistical properties (standard deviations) of the answers obtained from the adjustment
Why we want to use LSA

- Makes you a better surveyor, better measurement professional
- Understanding of equipment nuances
- Need to calibrate and maintain equipment
- Better survey procedures
- Teaches field personnel why they do what they do at your direction
Why we want to use LSA

- Error and Mistakes are not the same
  - Error is unavoidable and present in measurement
  - Surveyors are the experts in managing error
  - LSA allows us to anticipate error, fully manage, and even report it professionally and accurately.
  - Mistakes can be costly, inconvenient, painful
  - LSA is another tool to trap mistakes, even avoidable systematic error (which is a mistake as well)
Why we want to use LSA

- Stop being so certain
  - Absolute certainty in measurement is impossible
  - Develop an open mind about your results
    - That monument 0.04’ off?
    - These monuments not in a line?
    - Report your coordinates to 0.000X’?
How can we use LSA (wrongly)

- Within reason least squares can be used to obtain any desired answer.
- Procedure is specific and proven, but can also be abused, sometimes unwittingly.
- A solution can have questionable, marginal, little, or no value.
- Must assign legitimate weights to observations, remove mistakes, analyze, and correct.
How can we use LSA

- Identify the correct model for evaluating and adjusting (stochastic)
- Reduce the various measurements to common terms
- Assign initial estimates of error (weighting) and process estimated answers
- Run iterations, evaluate residuals, seek mistakes and systematic errors, converge, and done
How can we use LSA

- LSA is appropriate when:
  - There are redundant measurements
  - Only random errors are present

- LSA can:
  - Accommodate different precision of measurements
  - Give most probable results

- LSA requires correct weighting!
How can we use LSA

- Weighting:
  - Worth or reliability of a measurement
  - Measurements > precision < smaller std. deviation > weights
  - Equal weight to simple repeated measurement, not typical for complete survey
  - In correct adjustment of valid survey, measurements with higher weights should receive smaller corrections.
How can we use LSA

Statistical Summary:

- Error Factors
  - Individual for each type of measurement
  - Evaluates how well each type of data fits
  - .5 to 1.5
  - Can point to a particular data set for problems

- Total Error Factor (Reference Factor)
  - Should approach “1” for valid adjustment
How can we use LSA

- Chi Square Test:
  - Test for presence of only random error
  - Did the adjustment perform?
  - “Goodness-of-fit Test”
  - Upper bound fail warrants investigation and correction
  - Lower bound fail, judgement
How can we use LSA

- Residuals: Difference between observed and resulting value from adjustment
- In random errors, distribution (think bell curve) is that smaller residuals (think error) occur more frequently
- Best solution creates the least change from observations and allows statistical distribution of error
- Outliers identified (blunders?)
How can we use LSA

Applications

- Ability to report accuracy (confidence) in measurements and results
  - Control Surveys
  - Error Propagation
  - Monitoring
- Identify and remove mistakes and systematic errors
- Plan surveys to meet specified accuracies
- Record information
Moving forward with LSA

- Existing Software
  - Civil/3D
  - Carlson
  - TBC
  - LGO
  - Etc...
- StarNet
Moving forward with LSA

**StarNet** Tutorial through various type:

- 2D Traverse
- 3D Traverse
- 3D Geodetic (Grid)
- Resection & Sideshots
- Pre-Analysis
- GPS
- Leveling
Moving forward with LSA

- **StarNet** for your data
  - Convertors
    - GPS
      - Built-In?
    - Conventional
    - Leveling
  - Hand entry ?!
Moving forward with LSA

- Read the Manual – Read it again
- Start with what you are most familiar with, preferably a conventional survey.
- Assign errors from manufacturer where applicable and use experience, judgment, and guesses where not (Calibrate!)
- Consistency breeds understanding consistency
- Better measuring, better surveying
Questions or Comments?

Richard Maher, PLS
rmaher@kdmmeridian.com